Turbulent effect on self-organized structure formation in phase separation system

Okano laboratory
Phase separation: spinodal decomposition
Phase separation: spinodal decomposition

Multicomponent mixture phase separates by thermodynamic instability forming network structure.

Network structure

Self-organization structure
Self-organization structure

Honeycomb film (下村ら 特開2002-335949, Fuji Film)

What/How does the flow field (turbulent flow) affect a self-organization structure?
## Previous studies

### Theory & experiment

<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ruiz &amp; Nelson</td>
<td>1981</td>
<td>Energy cascade</td>
</tr>
<tr>
<td>Aronovitz &amp; Nelson</td>
<td>1984</td>
<td>Energy cascade, quench</td>
</tr>
<tr>
<td>Pine et al.</td>
<td>1984</td>
<td>Light scattering exp.</td>
</tr>
<tr>
<td>Tanaka</td>
<td>~ Present</td>
<td>Temp. quench, visoelastic</td>
</tr>
</tbody>
</table>

### Numerical simulation

<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Dimension</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berti et al.</td>
<td>2005</td>
<td>2D</td>
<td>Spectral</td>
</tr>
<tr>
<td>Kendon et al.</td>
<td>1999,2000,2001</td>
<td>3D</td>
<td>LBM</td>
</tr>
<tr>
<td>Perlekar et al.</td>
<td>2013</td>
<td>3D</td>
<td>LBM</td>
</tr>
</tbody>
</table>
Objective

• Relation between turbulent suppression by phase separation and energy cascade
• Analogy between self-organized structure by spinodal decomposition and coherent eddy structures in turbulence

Method:
• Three-dimensional simulation for homogeneous isotropic turbulence with phase separation

Horiuti & Takagi, PoF, 2005
Simplified model (Diffusion type)

• Order parameter (phase field):
  \[ \psi(\mathbf{r}, t) \equiv \left[ \rho_A(\mathbf{r}, t) - \rho_B(\mathbf{r}, t) \right]/\rho_0 \]

• Diffusion equation:
  \[ \frac{\partial \psi}{\partial t} + (\mathbf{v} \cdot \nabla)\psi = D \nabla^2 \psi \]

• Navier-Stokes equation:
  \[ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho_0} \nabla p' - \alpha \nabla \psi^2 \nabla \psi + \nu \nabla^2 \mathbf{v} + \mathbf{f} \]

  \( \mathbf{f} = 0 \): Decaying flow

See abstract
Numerical model
(phase separation under flow)

- Navier-Stokes equation

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho_0} \nabla p' - \xi^2 \nabla^2 \psi \nabla \psi + \nu \nabla^2 \mathbf{v} + \mathbf{f}
\]

- Continuity equation

\[
\nabla \cdot \mathbf{v} = 0
\]

- Cahn-Hilliard equation

\[
\frac{\partial \psi}{\partial t} + (\mathbf{v} \cdot \nabla)\psi = \Gamma \nabla^2 \mu
\]

\[
\mu = -\psi + \psi^3 - \xi^2 \nabla^2 \psi
\]

\[
\mathbf{f} = \begin{cases} 
Q\mathbf{v} & \text{Linear forcing} \\
0 & \text{Decaying flow} 
\end{cases}
\]

**Parameters**

\[
\nu = 2.5 \times 10^{-3} \quad (\nu/\Gamma = 0.1)
\]

\[
Q = 0.25 \quad (Re_\lambda \sim 75)
\]

\[
\xi = 1.0 \times 10^{-3}
\]

\[
\rho_0 = 1
\]
## Numerical conditions

<table>
<thead>
<tr>
<th>Analysis model</th>
<th>Homogeneous isotropic turbulence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixture</td>
<td>Binary mixture</td>
</tr>
<tr>
<td>Initial concentration</td>
<td>50%-50%</td>
</tr>
<tr>
<td>Domain size</td>
<td>$2\pi \times 2\pi \times 2\pi$</td>
</tr>
<tr>
<td>Boundary condition</td>
<td>Periodic on all the sides.</td>
</tr>
<tr>
<td>Grid points</td>
<td>$256^3$</td>
</tr>
<tr>
<td>Time increment</td>
<td>$\Delta t = 4.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>Initial condition of $\psi$</td>
<td>white noise</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Runs</th>
<th>Velocity forcing</th>
<th>Phase separation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>OFF (Stationary)</td>
<td>ON</td>
</tr>
<tr>
<td>2</td>
<td>Turbulence</td>
<td>OFF (Single component)</td>
</tr>
<tr>
<td>3</td>
<td>Turbulence</td>
<td>ON</td>
</tr>
</tbody>
</table>
Phase separation without flow field

Order parameter distribution on a $x$-$y$ plane

<table>
<thead>
<tr>
<th>Time ($t$)</th>
<th>Order Parameter Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0.6$</td>
<td>![Image at $t = 0.6$]</td>
</tr>
<tr>
<td>$t = 1.0$</td>
<td>![Image at $t = 1.0$]</td>
</tr>
<tr>
<td>$t = 2.0$</td>
<td>![Image at $t = 2.0$]</td>
</tr>
<tr>
<td>$t = 5.0$</td>
<td>![Image at $t = 5.0$]</td>
</tr>
<tr>
<td>$t = 8.0$</td>
<td>![Image at $t = 8.0$]</td>
</tr>
<tr>
<td>$t = 11.0$</td>
<td>![Image at $t = 11.0$]</td>
</tr>
</tbody>
</table>
Turbulent effect on phase separation

Phase separation was suppressed in presence of turbulence.

\[
L(t) = \frac{1}{\langle 1 - \psi^2 \rangle}
\]

Order parameter distribution on a \(x-y\) plane at \(t = 5.0\)
Turbulent effect on phase separation

Order parameter distribution on a $x$-$y$ plane at $t = 5.0$

**Coarsening length** $L$

$$L(t) = \frac{1}{\langle 1 - \psi^2 \rangle}$$

**Graphical Illustration**

- Without turbulence
- With turbulence

**Explanation**

- Stretching and folding by turbulence
- Breakup

**Diagram**

- Visual representation of phase separation with and without turbulence.
Suppression of vortex formation by phase separation

Vorticity ($z$-component) distribution on a $x$-$y$ plane at $t = 5.0$

Without phase separation effect

With phase separation effect

Vortex tube structure

(red line contour of $Q = 100$): second invariant of velocity gradient tensor

$$Q = \frac{1}{2} (|\Omega|^2 - |S|^2)$$

$\Omega_{ij}$: Vorticity tensor
$S_{ij}$: Strain rate tensor
Suppression of vortex formation by phase separation

Vorticity ($z$-component) distribution on a $x$-$y$ plane at $t = 5.0$

Energy spectrum at $t = 5.0$

Without phase separation effect

With phase separation effect

Energy in wide range of wave number decreased with phase separation.
Conclusions

- Turbulent vortex structure was more coarse in phase-separating mixture.
- Phase separation was suppressed and formed elongated structure in the presence of turbulence.

Self-organization structure can be selectively formed by turbulent field.